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An Element-by-Element Mild-Slope Model for Wave Propagation Studies

J.L.M. Fernandes†; M.A. Vaz dos Santos‡ and C.J.E.M. Fortes§

†Mechanical Department
IST, 1096, Lisbon, Portugal
leonel@dem.ist.utl.pt

‡Mechanics Department
FURG, 96201-900,
Rio Grande, RG, Brasil.
angela@dmc.furg.br

§Hydraulics Department
NPE-DHA, LNEC, Lisbon,
1700-066 Portugal
jfortes@lnec.pt

ABSTRACT

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In this paper, the EBE_MSE model for wave agitation and resonance studies in harbours and sheltered zones is described. The model is based on the element-by-element formulation applied to the mild-slope equation. In this formulation, there is no assembly of the global matrix of the system, which leads to important savings in storage requirements. For the solution of the system of equations the conjugate gradient iterative method (CGM), with a diagonal preconditioner is used. This technique was especially adapted to work with complex variables. In order to increase the performance of the iterative method, other preconditioners were implemented and their efficiency is evaluated. The EBE_MSE model was used to study the wave agitation inside the Saint Quay Portrieux harbour in France and the Lugar de Baixo marina in the Madeira island in Portugal. The results show that the element-by-element model leads to a significant reduction of the memory storage needed for the calculations. This advantage is especially useful for large scale problems. In what concerns the preconditioners implemented, it was concluded that all of them lead to a faster convergence of the basic iterative method.

ADITIONAL INDEX WORDS: *Wave propagation, mild-slope equation, finite element method, element-by-element formulation, iterative methods, preconditioners*.

INTRODUCTION

Numerical models for wave propagation that use the finite element method to solve the mild-slope equation, BERKHOFF (1972) are suitable for wave agitation and resonance studies in harbours, bays or coastal regions, in general. However, in order to comply with the condition relative to the minimum number of points per wavelength in the whole domain, their application leads to grids with a very large number of nodes and to very large system of equations. The computational effort and the storage required for the solution of such systems are so large that it may hamper the application of the models.

So, in order tackle that problem, MACEDO *et al*. (2001), developed an element-by-element model (EBE, HUGHES *et al*., 1987) that solves the long wave equation (or shallow water equation) by using the finite element method. In this EBE formulation, there is no need to assemble the system's global matrix; the system is stored at element level, instead. Because of this storage at the element level, an important reduction of the storage requirements is achieved. In parallel with this formulation, an iterative Diagonally Preconditioned Conjugate Gradient Method, HESTENES and STIEFFEL (1952), is used for the solution of the system of equations. This model, which is called EBE_SWE model, is particularly useful for very large regions due to the reduced storage needed.

However, an important limitation of this model is related to the fact that it is based on the long wave equation, and so, it is only valid for shallow waters. It is important to note that in the majority of wave propagation studies in harbours, the water depths vary from deep or intermediate water depths to shallow water depths.

So, some modifications were made to the EBE_SWE model in order to solve the mild-slope equation, a more general equation that is valid for shallow, intermediate and deep waters. The element-by-element formulation is used to solve the mild-slope equation. This modified model, named EBE_MSE, is valid for linear wave propagation over mild-slope bottoms, varying from deep to shallow waters.

In order to enhance the model's performance, some alternative methods of preconditioning were also implemented. They differ in terms of storage requirements, number of iterations, number of matrix-vector operations and execution time. We address the problem of accelerating convergence and compare the results with those of a standard diagonal preconditioning. Also important is to monitor how the number of iterations grows with the problem size.

In this paper, the EBE_MSE numerical model is described. First the basic equations are presented. Afterwards, the EBE formulation that uses the finite element model for the solution of the wave equation as well as the iterative conjugate gradient method is described. Several preconditioning techniques, which were tried to enhance the performance of the iterative method, are presented.

The EBE_MSE model is used to study the wave agitation in two real test cases: St. Quay Portrieux harbour (France) and Lugar de Baixo marina (Madeira Island, Portugal). The EBE_MSE results are compared with another solution of the mild-slope equation obtained by a direct method (skyline variant of Doolittle's method) to solve the system of equations. The model's performance as well as the preconditioners efficiency is presented and discussed.

BASIC MODEL EQUATION

The numerical model solves the mild-slope equation by using the finite element method. The mild-slope equation, for the surface elevation, H, and the boundary conditions are given by:

$$\frac{\partial}{\partial x_j}\left(cc_g \frac{\partial H}{\partial x_j}\right) + k^2 cc_g H = 0,\ (j = 1,2) \text{ in } V \tag{1}$$

$$\frac{\partial H}{\partial n} - ikH = f, \text{ in } S_1 \tag{2}$$

$$\frac{\partial H}{\partial n} = ik\alpha_{abs} H, \text{ in } S_2 \tag{3}$$

where c is the phase velocity and c_g is the group velocity, k is the wave number, is a value determined as a function of the incident wave characteristics (considering the generation-radiation condition in the open boundary S_1) and α_{abs} is the absorption coefficient in the solid boundaries S_2. While x_j (j=1,2) are the coordinates with respect to the reference system, n is a coordinate in the normal direction to boundaries S_1 and S_2. V is the domain, which will be analyzed.

The Finite Element Method is based on the weak formulation of Eqs. (1), (2) and (3). The weak formulation is obtained by weighting in the domain the residual of those equations by a test function. After some intermediate steps, the following equation, at the element level, is obtained:

$$\left[K^{(e)} - k^2 cc_g M^{(e)}\right] H^{(e)} = F^{(e)} \tag{4}$$

where

$$K^{(e)} = \int_V cc_g \frac{\partial N^T}{\partial x_j} \frac{\partial N}{\partial x_j} dV \,, \left(j = 1{,}2\right) \tag{5}$$

$$M^{(e)} = \int_V N^T \, N.dV \tag{6}$$

$$F^{(e)} = \int_{S_1} N^T \left(Nf^{(e)}\right) dS - ik \int_{S_2} \alpha_{abs} N^T N dS \tag{7}$$

In Eqs. (5) to (7), is the transpose of vector .

The system in Eq. (4) can be written in the following form:

$$A^{(e)} H^{(e)} = F^{(e)} \tag{8}$$

where

$$A^{(e)} = K^{(e)} - k^2 cc_g M^{(e)} \tag{9}$$

The global system of equations is implicitly obtained by adding all contributions at element level, Eq.(8).

PRECONDITIONED ITERATIVE METHOD IN A EBE FORMULATION

An element-by-element formulation is applied to Eq. (1). The solution of the system of equations is obtained with the gradient conjugate iterative method, HESTENES and STIEFFEL (1952). Several preconditioners are associated with this method in order to enhance the convergence of the method. These techniques consist in solving a supplementary system for the preconditioning matrix at each iteration. Further details will be presented in the following sections.

Element-by-Element Formulation

The element-by-element formulation (EBE) is a technique that works with the element equations and thus requires an amount of storage that is proportional to the number of elements. This means that the storage grows linearly with the number of equations. This contrasts favorably with the storage requirements for a skyline stored matrix that grows supralinearly with the problem size. For large problems, when memory is limited, skyline storage becomes prohibitive.

The EBE formulation is most adequate if the system of equations is solved by an iterative method because it requires a matrix-vector product plus an easy system solve in each iteration.

The off-diagonal coefficients of the complex symmetric element matrices are stored in an appropriate array GK while their diagonal coefficients are assembled into a global diagonal. For a triangular element symmetric matrix only three off-diagonal coefficients have to be stored per triangle. This seems to be a very economic way to store the coefficient matrix. For a mesh of triangles only this requires 3 times the number of elements plus the number of equations (for the assembled diagonal). As the number of triangles is about double the number of equations, the total storage for a symmetric matrix is about 7 times the number of equations.

Two important aspects must be considered in the computational implementation of EBE method with diagonal preconditioning: the matrix-vector product operation and the construction of the preconditioning matrix.

The matrix-vector product operation is carried out element-by-element-wise considering the off-diagonal element contribution added to the diagonal matrix multiplication.

Preconditioners

Preconditioning consists in implicitly modifying the original system of equations in such a way that the system matrix becomes better conditioned and thus the iterative process converges faster. Implicit preconditioning requires the solution of a companion system in each iteration whose matrix, the precondioning matrix, is an approximation of the original matrix A. This companion system should be quite easier to solve than the original one. We assume in what follows that matrix A is previously scaled by the diagonal. The preconditioners used, in this model, are:

1) Diagonal preconditioner $A^{-1} \approx I$, called Method EBE_MSE_PCG_I;
2) Preconditioner: $A^{-1} \approx (I + \omega U)\,(I + \omega L)$, called Method EBE_MSE_PCG_II;
3) Preconditioner $A^{-1} \approx (I + \omega_1 U)\,(I + \omega_2 LU)\,(I + \omega_1 L)$, denoted as Method EBE_MSE_PCG_III;
4) Polinomial preconditioner by a truncated Neumann series, $A^{-1} \approx I + B + B^2 + ... + B^{nt}$, denoted as Method EBE_MSE_PCG_IV,

where I, U and L are the diagonal, upper and lower part of the matrix A. The user specifies the parameters w, w_1 e w_2 and nt (which is the highest exponent in the Neumann series, nt=number of terms-1).

The first preconditioning matrix, denoted as Diagonal preconditioner - Method EBE_MSE_PCG_I, is built with the diagonal terms of the global coefficient matrix A and was the first preconditioner implemented in the original EBE_SWE model, MACEDO *et al*. (2001).

In the present work several other preconditioners were considered to approximate the inverse of A.

Since the matrix was scaled by the diagonal matrix I, we have the following additive decomposition of A:

$$A = I - L - U, \tag{10}$$

where L and U are the lower and upper matrices, respectively. That equation can be re-written as:

$$A = (I - L)\,(I - U) - LU \tag{11}$$

and, if we neglect the last term, an approximate factorization of A is given by:

$$A \approx (I - L)\,(I - U) \tag{12}$$

The inverse of this approximation is given by:

$$A^{-1} \approx (I - L)^{-1}\,(I - L)^{-1} \tag{13}$$

Considering the first order term in each factor, we obtain the following approximation for the inverse of A

$$A^{-1} \approx (I + U)\,(I + L) \tag{14}$$

HURDLE *et al*. (1989) have already used this preconditioner.

This expression can be further generalized by introducing an over-relaxation factor ω:

$$A^{-1} \approx (I + \omega U)\,(I + \omega L)(15)$$

This is the second option for preconditioning which was implemented in the model, and is designed as EBE_MSE_PCG_II. The parameter w is specified by the user and is usually in the range from 1.0 to 1.4.

One can also try to add a higher order correction term. In fact, by using the expression

$$A = (I - L)(I - U) - LU \quad (16)$$

we get

$$A = (I - L)(I - LU)(I - U) - LU^2 - L^2U + L^2U^2 \quad (17)$$

Neglecting the last three terms, which are of higher order, we get a new factorized approximation of A

$$A \approx (I - L)(I - LU)(I - U) \quad (18)$$

Now, taking into account the first order term of each factor, the following approximation of three factors for A^{-1} is obtained

$$A^{-1} \approx (I + U)(I + LU)(I + L) \quad (19)$$

and, again, the expression of the inverse matrix A can be generalized, by using two relaxation factors, ω_1 e ω_2:

$$A^{-1} \approx (I + \omega_1 U)(I + \omega_2 LU)(I + \omega_1 L) \quad (20)$$

This expression corresponds to the third option of preconditioning, denoted as EBE__MSE_PCG_III method. The user specifies the parameters ω_1 and ω_2. The ranges usually used are ω_1=1.2-1.4 and ω_2=1.8-2.0.

Finally, the fourth option, EBE__MSE_PCG_IV method, is a polynomial preconditioner by truncated Neumann series where:

$$A^{-1} \approx I + B + B^2 + ... + B^{nt}, \quad (21)$$

where the highest exponent nt (=number of terms-1) is specified by the user. The values of nt are less than 5 and usually, $2 \leq nt \leq 5$.

All four preconditioners were implemented the EBE_MSE model as well as the direct method. The direct method is a variant of the Doolittle LDL^T (L^T is the transpose of L matrix) factorization with skyline storage. In this way, we can compare the accuracy of the iterative solutions with the direct method as well as the time needed to obtain the solution.

NUMERICAL APPLICATIONS

The EBE_MSE model was applied to two real test cases: the Saint Quay-Portrieux harbour in France, FORTES (1993), and the Lugar de Baixo marina, FORTES *et al*. (2002).

In the first case, we study the propagation of long waves into the harbour while on the second test case short waves are tested. For both tests, the numerical results are presented and discussed.

The performance of the iterative method is evaluated as well as of the preconditioner techniques. Finally, we evaluate the convergence of the iterative method when low-resolution grids are considered. The number of iterations, the CPU time and the memory storage needed in the calculations are computed and discussed.

The results of the EBE_MSE model are the wave indexes, H/Ho, i.e., the relation between the wave height, H, at each point and the incident wave height, Ho. The results are compared with correspondent results obtained with the direct method implemented in the model.

Saint Quay-Portrieux Harbour

The Saint Quay-Portrieux harbour is located on the northern of France and it consists of an old fishing harbour and of a new one, which is also used as recreational harbour, Figure 1. In order to study the response of this new harbour to incident long waves, several resonance studies were performed, FORTES (1993). Here, we reproduce two of these resonance calculations in order to assess the accuracy of the EBE_MSE model and the preconditioners implemented. Further long waves of shorter period are also tested to evaluate the influence of grid resolution on the convergence of the iterative method.

Calculations and Results

The first test calculations are performed for two incident long waves with periods of 86 s and 360 s. Those periods are chosen because they lead to different resonance configurations of the St. Quay-Portrieux harbour. The incident wave direction is N-10°-E. The bathymetry adopted consists of several constant depth basins varying between 3.5 m to 10 m, as presented in Figure 1.

The domain considered was discretized with a finite element grid with 4889 nodes and 9305 elements, the grid spacing in both directions being 15 m. For the water depth ranges and periods considered, the minimum numbers of points per wavelength are 34 for T= 86 s and 148 for T= 380 s, respectively, which is significantly higher than the minimum number of points per wavelength needed to guarantee the accuracy of the solution.

The boundary conditions were the same as those used in FORTES (1993), i.e., a total reflection condition along the boundaries of the old and new harbours (E), a generation radiation condition at the entrance boundaries (A and B) and a radiation condition at the outgoing boundaries (C and D).

For those conditions, the EBE_MSE model with the diagonal preconditioner (EBE-MSE_PCG_I) is used to calculate the wave indexes in the entire domain, H/Ho. The same calculations were performed with the direct method. The memory required for the direct method skyline matrix is 214964 complex double words while the iterative requires about 68446 only. There is a significant storage reduction when the iterative method is used instead of the direct one. Conversely, the CPU time associated with the direct method is only 0.3 s while with the iterative method is 2.7 s, for T= 86 s.

Figure 2 and Figure 3 present the numerical results of EBE_MSE, for T=86 s and 360 s. Those figures show that there is a significant increase of the wave indexes inside the new harbour, for both periods, and so resonance problems do occur. In the old harbour, the wave indexes are smaller and no resonance problems occur. As in FORTES (1993), the periods tested lead to different resonance configurations. In fact, for T= 86 s, the new harbour basin oscillates with a node localized at the central part of the basin, while for T= 360 s, it oscillates with a node located at the entrance of the new harbour basin. Finally, for T = 86 s and 380 s, the wave indexes in the new harbour are less than 1.6 and 4.0, respectively. A similar behavior was found in FORTES (1993).

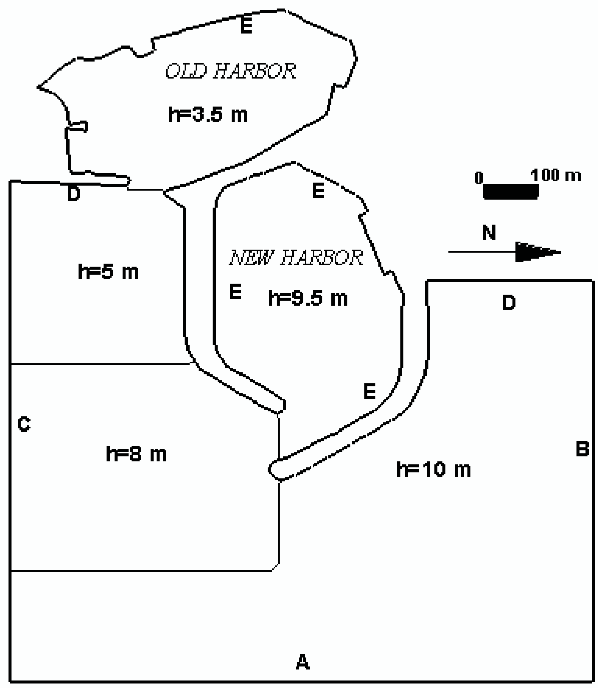


Figure 1. Saint Quay-Portrieux harbour. Geometry, domain and bathymetry considered.

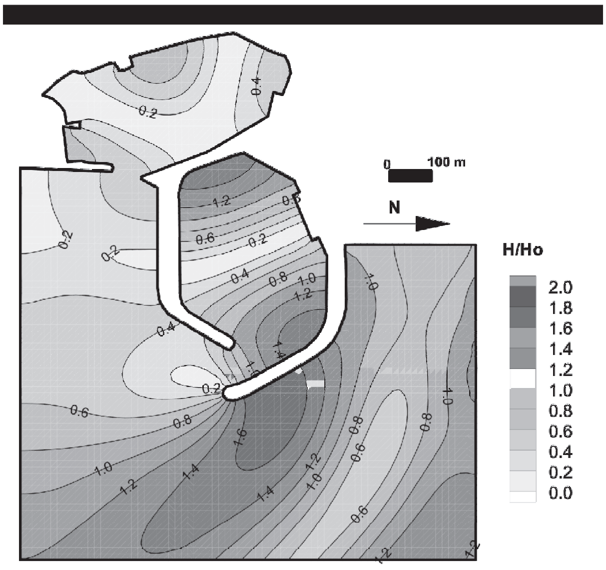


Figure 2. T=86 s. Wave height indexes.

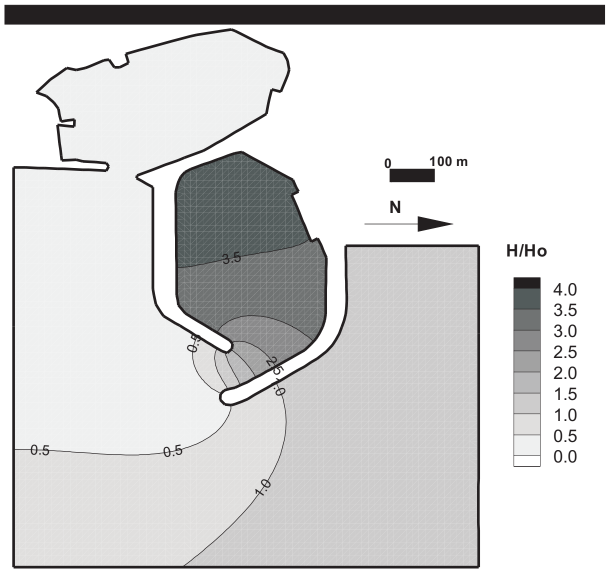


Figure 3. T=380 s. Wave height indexes.

The second set of calculations with the EBE_MSE aim to evaluate the efficiency of the four preconditioners implemented. For each preconditioner, the solution was calculated considering the same above conditions and a T= 86 s period. Different values of the parameters associated to each preconditioner were also tested. For each test, the CPU time, the number of iterations and the equivalent number of matrix-vector multiplications, MVM, are presented at Table 1.

In general, all four preconditioners lead to a good performance of the iterative method, i.e., there are no convergence problems. The CPU time associated with these preconditioners is always greater than the one associated to the direct method (0.3 s).

Table 1. *Number of iterations, MVM and CPU time.*

Method	Parameters				N. Iter.	MVM	CPU (s)
	ω	ω_1	ω_2	nt			
I	-	-	-	-	506	507	2.7
II	1.0	-	-	-	302	606	4.1
II	1.1	-	-	-	316	634	4.4
II	1.2	-	-	-	327	656	4.5
II	1.3	-	-	-	324	650	4.4
II	1.4				335	672	5.1
II	1.5	-	-	-	333	668	4.6
II	1.6	-	-	-	335	672	4.6
II	1.7	-	-	-	344	690	4.7
II	1.8				367	736	5.1
II	1.9				367	736	5.1
II	2.0	-	-	-	402	806	5.7
III	-	1.2	1.8	-	316	951	6.8
III	-	1.3	1.8	-	313	942	6.6
III	-	1.4	1.8	-	336	1011	7.2
III	-	1.2	1.9	-	315	948	6.7
III	-	1.3	1.9	-	319	960	6.8
III	-	1.4	1.9	-	335	1008	7.2
III		1.2	2.0		317	954	6.8
III		1.3	2.0		316	951	6.7
III		1.4	2.0		337	1014	7.2
IV	-	-	-	1	309	620	2.7
IV	-	-	-	2	346	1041	4.5
IV				4	257	1290	5.3
IV				6	244	1715	7.0
IV	-	-	-	8	214	1935	7.9
IV	-	-	-	10	217	2398	9.6

The more efficient preconditioners, in terms of CPU time, are the EBE_MSE_PCG_IV (nt=1) and the EBE_MSE_PCG_I (diagonal preconditioner). The EBE_MSE_PCG_IV (nt=1) is the preconditioner that leads to a lower number of iterations. Otherwise, the diagonal preconditioner is not associated to the lower number of iterations. Anyway, the differences between the CPU time of the other preconditioners II and III are not significant. The preconditioner II leads to the solution in 5 s (on average) and preconditioner III in 6 s (on average). The most adequate parameters associated to the preconditioners, in terms of CPU time, are: EBE_MSE_PCG_II, ω=1, EBE_MSE_PCG_III, ω_1=1.3 e ω_2=1.8 and EBE_MSE_PCG_IV, nt=1.

Finally, the EBE_MSE_PCG_I model is applied considering different long wave periods (from 86 to 25 s), in order to evaluate the influence of the grid resolution (number of points per wavelength) in the convergence of the iterative method. Notice that for a lower number of points per wavelength, the convergence of the iterative method is difficult and is not guaranteed *a priori*. Table 2 presents the minimum number of points per wavelength, the number of iterations and equivalent number of matrix-vector multiplications, MVM and the CPU times associated to EBE_MSE_PCG_I model and to direct method. The error (Error) in relation to the solution obtained with the direct method that was obtained with EBE_MSE_PCG_I model is also calculated.

From Table 2, we can see that the number of iterations, CPU time and MVM increase as the number of points per wavelength decreases (wave period decrease). The larger number of iterations and CPU times occur when the number of points per wavelength is less than 15.

Table 2. *Number of iterations, MVM, CPU times for Iterative and Direct methods and Error.*

T(s)	N.Point/L	EBE-MSE_PCG_I (Diagonal preconditioner)				
		N. Iter.	MVM	CPU (s) Iterative	CPU (s) Direct	Error
86	34	506	507	2.7	0.3	3.07E-06
60	23	677	678	3.1	0.3	1.32E-08
50	20	696	697	3.8	0.3	1.59E-07
40	16	817	818	4.3	0.3	8.52E-07
35	14	972	973	5.3	0.3	1.31E-04
30	12	1126	1127	5.3	0.3	2.04E-07
25	10	1071	1072	5.8	0.3	5.60E-03

Lugar de Baixo marina

The Lugar de Baixo marina is located in Madeira Island, in Portugal, Figure 4, between the Ponta do Sol and Ribeira Brava regions.

FORTES *et al*. (2002) performed a study to evaluate the sheltering of this marina under the penetration of short waves. Here, we reproduce only two incident wave conditions in order to evaluate the performance of the EBE_MSE model and the preconditioners implemented. Notice that, for the periods considered, the medium number of points per wavelength is 21.

Calculations and Results

The first test calculations are performed for an incident wave with a period of 14 s and a wave direction of SE. The domain as well as the bathymetry considered are presented in Figure 5. The sea water level corresponds to 1.4 m (CD). The domain is discretized with a finite element grid with 79034 nodes and 156416 elements, the grid spacing being 2 m.

For this period, the minimum number of points per wavelength (L) at the lowest water depth is 12.

The boundary conditions, Figure 5, are a generation radiation condition at the entrance boundaries (B and C) and a radiation condition at the outgoing boundary (A). At the other boundaries, a partial reflection condition is imposed. The calculation of the reflection coefficients depends on the wave and the stretch characteristics, and is based on the FORTES SEELIG and AHRENS (1995) method, FORTES *et al*. (2002).

For the above conditions, the EBE_MSE model calculates the wave indexes at each point of the domain, H/Ho. Figure 6 presents the numerical results of EBE_MSE for T=14 s.

As we can see, for T= 14 s, there is significant reduction of the wave indexes as the wave propagates inside the marina. The values of the wave indexes at the entrance for the marina are higher due to the multiple reflections in the coast, but they decrease very much inside the marina. The maximum value of H/Ho inside the marina is 0.35.

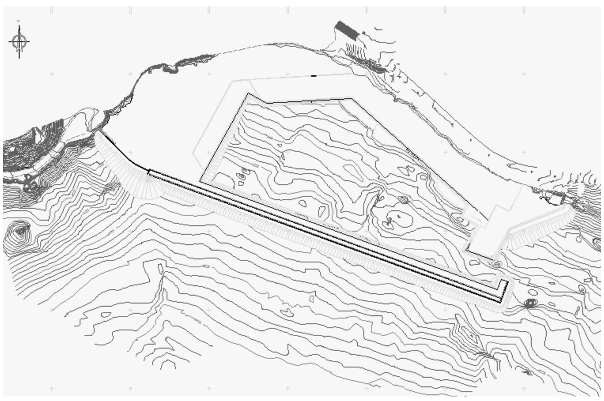
The next calculations with the EBE_MSE aim to evaluate the efficiency of the four preconditioners implemented, when short waves are considered. So, for T=14 s, several tests are performed with the four preconditioners, considering the most adequate parameters found for the St. Quay-Portrieux calculations. The CPU time, the number of iterations and the equivalent number of matrix-vector multiplications, MVM, are calculated, see Table 3.

As we can see, for all preconditioners considered (except for EBE_MSE_PCG_II, nt= 2.0), the iterative method is convergent. The CPU time associated with these preconditioners is always higher than the one associated to a direct method (78 s).

The more efficient preconditioners, in terms of CPU time, are the EBE_MSE_PCG_I (diagonal preconditioner), followed by the EBE_MSE_PCG_IV (nt=1), as concluded for the previous St.Quay-Portrieux test. In the actual Lugar de Baixo example, the preconditioner that leads to a smaller number of iterations is EBE_MSE_PCG_III (ω_1=1.3, ω_2=1.8 or ω_2=2.0) but at the expense of more work per iteration. Notice in this case the differences between the times of other preconditioners relative to the diagonal one. It is almost the same for the preconditioner II with ω = 1.0, and 50% higher for the preconditioner III with ω_1=1.3, ω_2=1.8. The others are quite worse than those.

Comparing the performance of the direct and of the iterative method, it is obvious that the direct method yields the solution

a)



b)

Figure 4. a) Lugar de Baixo region. b) Marina scheme.

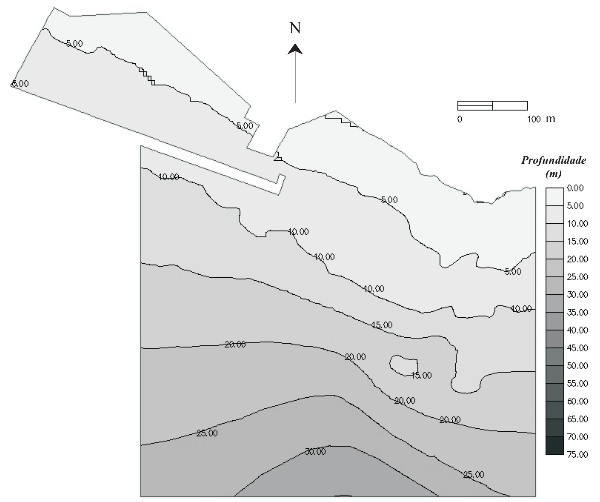


Figure 5. Domain. Bathymetry and stretches of the boundary.

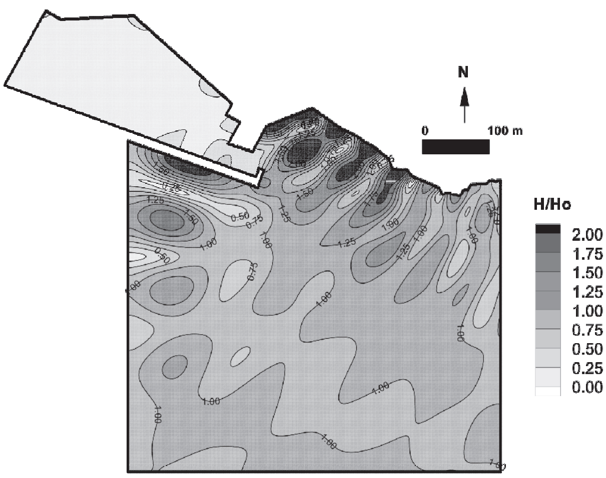


Figure 6. Results from EBE_MSE for T= 14 s.

Table 3. *T= 14 s. Number of iterations, MVM and CPU time.*

Method	Parameters				N. Iter.	MVM	CPU (s)
	ω	ω_1	ω_2	nt			
I	-	-	-	-	1373	1374	272.7
II	1.0	-	-	-	737	1476	287.7
II	1.1	-	-	-	763	1528	308.1
II	1.2	-	-	-	720	1442	292.9
II	1.3	-	-	-	955	1912	387.5
II	1.4				1068	2138	434.0
II	1.5	-	-	-	1057	2116	428.6
II	1.6	-	-	-	1038	2078	424.3
II	1.7	-	-	-	1035	2072	424.0
II	1.8				1183	2368	477.8
II	1.9				1221	2444	493.8
II	2.0	-	-	-	N/C	N/C	N/C
III	-	1.2	1.8	-	766	2301	480.2
III	-	1.3	1.8	-	699	2100	431.9
III	-	1.4	1.8	-	805	2418	514.6
III	-	1.2	1.9	-	738	2217	461.6
III	-	1.3	1.9	-	733	2202	453.3
III	-	1.4	1.9	-	769	2310	481.7
III		1.2	2.0		738	2217	462.9
III		1.3	2.0		699	2100	433.0
III		1.4	2.0		727	2184	455.6
IV	-	-	-	1	1064	2130	295.2
IV	-	-	-	2	793	2382	311.0
IV				4	646	3235	402.3
IV				6	571	4004	488.3
IV	-	-	-	8	509	4590	548.5
IV	-	-	-	10	452	4983	591.8

faster. However, the memory required for the direct method skyline matrix is 19,895,402 complex double words while the iterative requires about 1,101,520 only. This is a very important aspect, since it implies that the applicability of the iterative method to very large regions is possible with a low memory storage demand.

CONCLUSIONS

In this paper, an element-by-element mild-slope model for wave propagation and deformation of waves in harbours is described. The model is based upon the element-by-element formulation, HUGHES *et al*. (1987), applied to solve the mild-slope equation, BERKHOFF (1972). For the solution of the system of equations, a conjugate gradient iterative method with several preconditioning options is used. The model is applied to the real test cases of wave propagation in harbours: Saint Quay-Portrieux harbour, FORTES (1993) and Lugar de Baixo marina, FORTES *et al.* (2002).

The results show that the element-by-element model leads to a significant reduction of the memory storage needed to the calculations. This advantage is especially useful for large scale problems.

In relation to the preconditioners implemented, the conclusion was that all of them lead to a faster convergence of the base iterative method. The diagonal preconditioner (EBE_MSE_PCG_I method) and the EBE_MSE_PCG_IV with nt=1.0 are the most efficient ones, for both long wave and short wave studies.

The range of points per wavelength below 15 points is of critical importance in large scale applications and it is also the most demanding in terms of number of matrix-vector operations. The experiences performed suggest that other preconditioners should be evaluated in order to accelerate the convergence of the iterative method.

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