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An Improved General Dual Reciprocity Boundary Element Model for Wave Scattering by a Truncated Shoal

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ABSTRACT

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There are many numerical methods to model wave scattering by an island or a submerged shoal. However, for a submerged truncated shoal, numerical solution has rarely been reported before. In this paper, the wave model based on the general dual reciprocity boundary element method (GDRBEM) is extended to solve the mild-slope equation for a truncated shoal. The computational domain, which is generally divided into two sub-domains in a conventional GDRBEM model, is divided into three sub-domains in this study to separate the inner region with constant depth as an individual sub-domain. Using the current GDRBEM model, wave amplifications by a truncated conical shoal for various incident waves are calculated and compared with both the approximate analytic solutions and the previous GDRBEM solutions. It is found that the current GDRBEM model provides a much better comparison with the approximate analytic solutions than its predecessor. This improved GDRBEM model is then used to investigate the depth effects on the maximum wave amplification. The maximal wave amplification decreases when the submergence increases.

ADDITIONAL INDEX WORDS: *Water waves, the mild-slope equation*, *wave amplification.*

INTRODUCTION

The mild-slope equation (MSE) was derived by BERKHOFF (1972) where he demonstrated that, if the bottom slope is 'mild', a three-dimensional wave problem could be well approximated by a two-dimensional single equation. Since then, the MSE has proved a useful tool for a wide range of water wave problems.

Several numerical approaches have been developed to solve the MSE. For example, BETTESS and ZIENKIEWICZ (1977) and HOUSTON (1981) developed the so-called hybrid method for the MSE, in which an infinite computational domain is divided into two: an outer region where infinite elements or eigen-functions can be adopted and an inner region where finite element or difference techniques can be used to obtain solutions. Most of the subsequent work differs only on the treatment of the outer domain.

However, as the MSE is of the elliptical type, the amount of data and hence the size of the coefficient matrix become very large when finite element and hybrid element methods are used, and therefore the computation becomes very expensive for large domain simulation as shown by HOUSTON (1981).

The boundary element method (BEM) only requires a discretization on the boundary of a computational domain and it is popular in solving wave propagation problems with constant water depth (AU and BREBBIA, 1983). However, when the water depth becomes a variable, the conventional BEM seems to be powerless because that a domain integral arises and the domain has to be discretized, which destroys the computational advantage of the conventional BEM.

Employing the dual reciprocity boundary element method (DRBEM), which was first proposed by NARDINI and BREBBIA (1982), ZHU (1993a) presented a DRBEM wave model to solve the MSE. It is shown that the DRBEM model possesses a great advantage in numerical efficiency over hybrid elements method, in terms of both computational time and computer memory required. For example, for the case of Homma's island (HOMMA, 1950) with incident wave of period =120 sec, HOUSTON (1981) had to carry out his calculation with 10,560 elements on the half of a symmetrical domain. For a general domain without symmetry to be utilized, he would have to use 21,120 elements which would lead to a linear system with 21,600 real equations. However, using only 60 quadratic boundary elements and 192 internal collocation points for all the calculations covering the full domain, ZHU (1993a) has obtained an accurate result comparing with the results presented by JONSSON *et al*. (1976) and HOUSTON (1981).

Following the idea of RANGOGNI (1988), where varying water depth is approximated with a perturbed constant depth, POULIN (1997) proposed another DRBEM model to solve the MSE. This new model leads to a significant reduction of the number of unknowns in the resulting system of equations. However, since the governing equation solved by POULIN (1997) is only an approximate form of the MSE, the accuracy of solutions, as shown in the verification of the model for Homma's island, is satisfactory only for seabed geometry with a ratio of the water depth in the deep ocean to that along the coastal vertical wall less than 3. For Homma's original island with the depth ratio of 9, the accuracy is acceptable only for very long waves. Hence, this perturbation DRBEM model is not as accurate as the DRBEM model proposed by ZHU (1993a).

Then, considering that there is a restriction that the water depth is always nonzero in the DRBEM model (ZHU, 1993a), which narrows the range of application of the DRBEM model in comparison with its counterparts of hybrid finite elements method, ZHU *et al*. (2000) (see LIU, 2001 also) developed an extension of Zhu's DRBEM model to a general DRBEM model (GDRBEM) for cases where zero-water-depth coastlines are allowed.

In this paper, we consider to modify the GDRBEM model for wave scattering by a submerged truncated shoal. For such kind of truncated shoals, numerical solutions from both hybrid finite elements and DRBEM/GDRBEM solution have not yet been reported. Although the similar problem was once solved by CHAMBERLAIN and PORTER (1999) by using a standard error-checking Runge-Kutta method, their truncated shoal is restricted the axi-symmetrical topography that permits the utilization of cylindrical coordinates and the technique of variable separation.

It is clear that, for a truncated shoal, the bed slope along the boundary of the shoal top is discontinuous. Some attention has

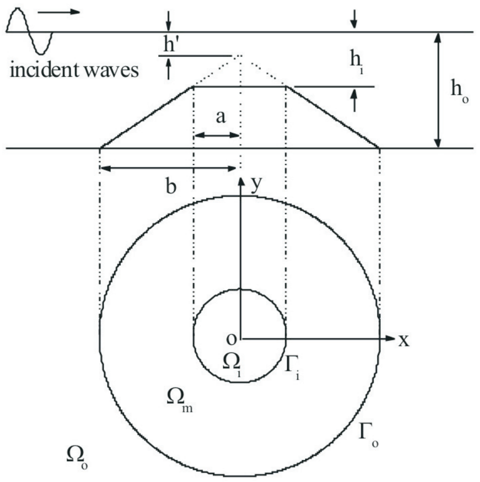

Figure 1. A definition sketch of a submerged truncated conical shoal.

been paid to this discontinuity. According to PORTER and STAZIKER (1995) and CHAMBERLAIN and PORTER (1999), to ensure the continuity of mass flow for waves crossing an ocean bottom with a discontinuity of the bed slope, the MSE should be replaced by the extended MSE or the modified MSE together with an interfacial jump condition being imposed there.

The aim of this paper is not to study the influence of this discontinuity on the governing equation. Rather, we shall return to the conventional MSE and study the influence of the discontinuity on the numerical accuracy of the GDRBEM model. As we have known, in finite element and finite difference method, the general way to resolve the influence of the topographic discontinuity is to refine the local meshes. However, we shall show in this paper that, this kind of refinement technique is not valid in the GDRBEM, and in fact a good alternative is to divide the computation domain into sub-domains along the curve of bed-slope discontinuity.

THE GOVERNING EQUATION

Under a Cartesian coordinate system in which x and y denote a pair of orthogonal horizontal coordinates and z denotes the vertical coordinate measured positively upwards from the undisturbed free surface, the incident wave potential of a train of monochromatic waves propagating along the positive x-axis over a submerged truncated shoal with variable water depth $h(x, y)$ can be expressed

$$\phi^I(x,y) = -\frac{ig}{\omega} A e^{ik_o x} \tag{1}$$

where g is the gravitational acceleration, A the incident wave amplitude, the angular frequency and the wave number with respect to the constant water depth outside the shoal.

According to BERKHOFF (1972), the total wave potential, $\phi(x,y)$, by which the refraction of these waves due to the topographic change of seabed and the existence of the shoal are described, satisfies the MSE

$$\nabla\cdot(G\nabla\phi)+k^2G\phi=0, \tag{2}$$

in which and

$$G(x,y)=\frac{g}{2k}\tanh kh\left(1+\frac{2kh}{\sinh 2kh}\right) \tag{3}$$

Where k is the local wave number determined by the dispersion relation.

The differential equation (2) is defined on an infinite computational domain. As shown in Figure 1, such a computational domain can be divided into three sub-domains Ω_i, Ω_m and Ω_o with Ω_i denoting the finite inner region with constant water depth h_i Ω_m denoting the finite middle region with variable water depth and Ω_o denoting the infinite outer region with constant water depth h_o. Since the governing equations in these regions will be transformed into different form, they should be discussed separately.

THE EQUIVALENT INTEGRAL EQUATIONS

Generally, in the DRBEM (NARDINI and BREBBIA, 1982), to minimize the interpolation error, it is preferable that the right-hand side of the governing equation be kept as simple as possible. On the other hand, the simplicity of the main differential operator should be taken into account in order to obtain the corresponding particular solution analytically. Particularly, for the problem of wave refraction and diffraction, in order to convert the original governing equation defined in the whole infinite region into an equivalent integral equation in a finite region, we generally need utilize the far-field radiation condition (SOMMERFELD, 1949) to eliminate the integral along the infinite circle (see LIU, 2001, p.33). Therefore it is convenient to choose the Helmholtz operator as the main differential operator, see ZHU *et al*. (2000) and LIU (2001). For the same reason here, we deliberately rewrite the governing equation (2) as

$$\nabla^2(G\phi)+k_o^2(G\phi)=R \tag{4}$$

where

$$R=(k_o^2-k^2)G\phi+\nabla G\cdot\nabla\phi+\phi\nabla^2 G \tag{5}$$

Let $\phi^*(X,\xi)=i/4H_0^{(1)}(k_o\rho)$ be the Hankel function of the first kind of zero order with $\rho=|X-\xi|$ being the distance between a source point ξ and a field point $X=(x,y)$ It is well-known that is the fundamental solution of the Helmholtz equation

$$\nabla^2\phi+k_o^2\phi=-\delta(X-\xi) \tag{6}$$

In the inner region Ω_i, since that $G(x,y)\equiv const$, equation (4) is equivalent to

$$G(\nabla^2\phi+k_o^2\phi)=R_i \tag{7}$$

Where $R_i=(k_o^2-k_i^2)G\phi$ with k_i being the local wave number in Ω_i. Multiplying both sides of equation (7) by $\phi^*(X,\xi)$ and using Green's second identity, one can rewrite equation (7) as

$$c_\xi^{(i)}G(\xi)\phi(\xi)+\int_{\Gamma_i} G\left(\phi\frac{\partial\phi^*}{\partial\vec{n}_i}-\phi^*\frac{\partial\phi}{\partial\vec{n}_i}\right)d\Gamma=-\int_{\Omega_i} R_i\phi^* d\Omega \tag{8}$$

where $c_\xi^{(i)}$ is a geometric parameter with respect to Ω_i $c_\xi^{(i)}=1$ for $\xi\in\Omega_i$, $c_\xi^{(i)}=\alpha^{(i)}(\xi)/(2\pi)$ for $\xi\in\Gamma_i$ and $c_\xi^{(i)}=0$ for $\xi\in\Omega_m+\Gamma_o+\Omega_o$ respectively, with $\alpha^{(i)}(\xi)$ being the internal angle of the boundary at point ξ. In the middle region Ω_m, for any fixed source point ξ,

$$c_\xi^{(m)}G(\xi)\phi(\xi)-\int_{\Gamma_i^+ +\Gamma_o^-}\frac{\partial G}{\partial\vec{n}_m}\phi\phi^* d\Gamma+\int_{\Gamma_i+\Gamma_o} G\left(\phi\frac{\partial\phi^*}{\partial\vec{n}_m}-\phi^*\frac{\partial\phi}{\partial\vec{n}_m}\right)d\Gamma \tag{9}$$
$$=-\int_{\Omega_m} R\phi^* d\Omega$$

where $c_\xi^{(m)}$ is a geometric parameter with respect to Ω_m: $c_\xi^{(m)}=1$ for $\xi\in\Omega_m$, $c_\xi^{(m)}=\alpha^{(m)}(\xi)/(2\pi)$ for $\xi\in\Gamma_i+\Gamma_o$ and $c_\xi^{(m)}=0$ for $\xi\in\Omega_i+\Omega_o$, respectively, with $\alpha^{(m)}(\xi)$ being the internal angle of the boundary at point ξ.

In the outer region Ω_o $G(x,y)\equiv const$, equation (4) is equivalent to the Helmholtz equation

$$G(\nabla^2\phi_s+k_o^2\phi_s)=0 \tag{10}$$

where Φ_s is the scattered wave. Multiplying both sides of equation (10) by $\phi^*(X,\xi)$ and using the far-field radiation condition (SOMMERFELD, 1949), one can rewrite (10) as

$$c_\xi^{(o)}G(\xi)\phi_s(\xi)+\int_{\Gamma_o} G\left(\phi_s\frac{\partial\phi_*}{\partial\vec{n}_o}-\phi^*\frac{\partial\phi_s}{\partial\vec{n}_o}\right)d\Gamma=0 \tag{11}$$

were $\vec{n}$ is the outward normal unit vector of the outer domain Ω_o and $c_\xi^{(o)}$ is also a geometric parameter with respect to Ω_o : $c_\xi^{(o)}=1$ for $\xi \epsilon \Omega_o$, $c_\xi^{(0)}=\alpha^{(0)}(\xi)(2\pi)$ for $\xi \epsilon \Omega_o$, and $c_\xi^{(0)}=0$ for $\xi \epsilon \Omega_i+\Omega_m+\Gamma_i$ respectively, with $\alpha^{(0)}(\xi)$ being the internal angle of the boundary at point ξ.

It is noted that, along the boundary Γ_i between Ω_i and Ω_m , we have

$$\frac{\partial\phi_*}{\partial\vec{n}_i}=-\frac{\partial\phi_*}{\partial\vec{n}_m} \quad \frac{\partial\phi}{\partial\vec{n}_i}=-\frac{\partial\phi}{\partial\vec{n}_m} \tag{12}$$

substituting equation (12) into equation (8) yields

$$\int_{\Gamma_i} G\left(\phi\frac{\partial\phi_*}{\partial\vec{n}_m}-\phi_*\frac{\partial\phi}{\partial\vec{n}_m}\right)d\Gamma=c_\xi^{(i)}G(\xi)\phi(\xi)+\int_{\Omega_i} R_i\phi_* d\Omega \tag{13}$$

And the continuity of the wave potential and flux across the common boundary Γ_m demands

$$\phi_s=\phi-\phi^I \ , \frac{\partial\phi_s}{\partial\vec{n}_o}=\frac{\partial\phi^I}{\partial\vec{n}_m}-\frac{\partial\phi}{\partial\vec{n}_m}. \tag{14}$$

Substituting equation (14) into equation (11) yields

$$\int_{\Gamma_o} G\left(\phi\frac{\partial\phi_*}{\partial\vec{n}_m}-\phi_*\frac{\partial\phi}{\partial\vec{n}_m}\right)d\Gamma=c_\xi^{(o)}G(\xi)(\phi(\xi)-\phi^I(\xi))$$
$$+\int_{\Gamma_o} G\left(\phi^I\frac{\partial\phi_*}{\partial\vec{n}_m}-\phi_*\frac{\partial\phi^I}{\partial\vec{n}_m}\right)d\Gamma. \tag{15}$$

By putting equations (13) and (15) into equation (9), we have

$$G(\xi)\phi(\xi)-\int_{\Gamma_i^++\Gamma_o^-}\frac{\partial G}{\partial\vec{n}_m}\phi\phi_* d\Gamma$$
$$=c_\xi^{(o)}G(\xi)\phi^I(\xi)-\int_{\Gamma_o} G\left(\phi^I q^*-\phi^* q^I\right)d\Gamma \tag{16}$$
$$-\int_{\Omega_m} R\phi_* d\Omega-\int_{\Omega_i} R_i\phi_* d\Omega \ , \quad \xi\in\Omega_i+\Omega_m+\Gamma_i+\Gamma_o$$

where $q_*=\partial\phi_*/\partial\vec{n}_m$, $q^I=\partial\phi^I/\partial\vec{n}_m$, and $c_\xi^{(i)}+c_\xi^{(m)}+c_\xi^{(o)}\equiv 1$ is used.

Up to this stage, the original governing equation (2) defined on the infinite domain $\Omega=\Omega_i+\Omega_m+\Omega_o+\Gamma_i+\Gamma_o$ has been transformed into the above equivalent integral equation (16) defined on a finite domain $\Omega_i+\Omega_m+\Gamma_i+\Gamma_o$. That is, all the source points ξ and field points X are now restricted to the inner region Ω_i and the middle region Ω_m as well as their boundaries Γ_i and Γ_o. This has greatly simplified the solution procedure in comparison with the hybrid finite element approaches (HOUSTON, 1981). All but two terms in equation (16) require information only on the boundaries Γ_i and Γ_o. To eliminate the two domain integrals, which involve the unknown function $\Phi(X)$ and its gradient $\nabla\phi$, the GDRBEM will be employed in the next section.

THE DUAL RECIPROCITY BOUNDARY ELEMENT

Considering that the main differential operators in governing equation is the Helmholtz operator, we shall follow ZHU *et al.* (2000) to choose the interpolation functions to be the following radial basis functions:

$$f_j(X)=1+\|X-X_j\|^2+\|X-X_j\|^3 \tag{17}$$

Thus, the right-hand side term in equation (4) is expanded as

$$R(X)=\sum_{j=1}^{N}\alpha_j f_j(X) \tag{18}$$

where $N=l+p+m+n$, and α_j are the coefficients to be determined with the collocation method by demanding the satisfaction of N equations

$$R(X)\Big|_{X_i}=\sum_{j=1}^{N}\alpha_j f_j(X_i)\ , I=l,...,N, \tag{19}$$

at l interior collocation points within Ω_i, p boundary points on Γ_i, m interior points within Ω_m and n boundary points on Γ_o.

System (19) can also be expressed in matrix form:

$$R=F\alpha \tag{20}$$

thus we have

$$\alpha=F^{-1}R \tag{21}$$

It is noted that the existence and the recursion formulae of the particular solution $\hat{\phi}_j(X)$ to the following equation

$$\nabla^2\phi+k_o^2\phi=f_j(X) \tag{22}$$

for a given f_j have been given by ZHU (1993b). The first domain integral in the right-hand side of equation (16) therefore becomes

$$\int_{\Omega_m} R\phi^* d\Omega\approx\sum_{j=1}^{N}\alpha_j\int_{\Omega_m} f_j(X)\phi^* d\Omega$$
$$=\sum_{j=1}^{N}\alpha_j\int_{\Omega_m}[\nabla^2\hat{\phi}_j+k_o^2\hat{\phi}_j]\phi^* d\Omega \tag{23}$$
$$=\sum_{j=1}^{N}\alpha_j[-c_\xi^{(m)}\hat{\phi}_j(\xi)-\int_{\Gamma_i+\Gamma_o}(\hat{\phi}_j q_*-\phi^*\hat{q}_j)d\Gamma]$$

which involves boundary integrals only. Similarly, the second domain integral in the right-hand side of equation (16) becomes

$$\int_{\Omega_i} R_i\phi^* d\Omega\approx\sum_{j=1}^{N}\alpha_j[-c_\xi^{(i)}\hat{\phi}_j(\xi)+\int_{\Gamma_i}(\hat{\phi}_j q_*-\phi^*\hat{q}_j)d\Gamma] \tag{24}$$

substituting equations (23) and (24) into equation (16) yields

$$G(\xi)\phi(\xi)-\int_{\Gamma_i^++\Gamma_o^-}\frac{\partial G}{\partial\vec{n}_m}\phi\phi^* d\Gamma$$
$$=c_\xi^{(o)}G(\xi)\phi^I(\xi)-\int_{\Gamma_o} G\left(\phi^I q^*-\phi^* q^I\right)d\Gamma \tag{25}$$
$$+\sum_{j=1}^{N}\alpha_j[c_\xi^{(i,m)}\hat{\phi}_j(\xi)+\int_{\Gamma_o}(\hat{\phi}_j q^*-\phi^*\hat{q}_j)d\Gamma]$$

for $\xi \epsilon \Omega_i+\Omega_n+\Gamma_i+\Gamma_o$, where $c_\xi^{(i,m)}=c_\xi^{(i)}+c_\xi^{(m)}$, that is, $c_\xi^{(i,m)}=1$ for $\xi\in\Omega_i+\Omega_m+\Gamma_i$, $c_\xi^{(i,m)}=\alpha^{(m)}(\xi)/(2\pi)$ for $\xi\in\Gamma_o$ and $c_\xi^{(i,m)}=0$ for $\xi\in\Omega_o$ Equation (25) involves boundary integrals only and after appropriate discretization, a linear system of algebraic equations involving the unknown function Ø on N points can be established. The details of a similar system for constant boundary elements can be found in the Appendix A in ZHU *et al.* (2000) or in LIU (2001).

It is worth indicating that if we still adopt the conventional GDRBEM model (ZHU *et al.*, 2000 and LIU, 2001) to model the wave scattering by a submerged truncated shoal, that is, we ignore the discontinuity of the bed slope along the top boundary and divide the computational domain into two sub-domains only along the footline , then we shall have the final integral equation as follows

$$G(\xi)\phi(\xi)-\int_{\Gamma_o^-}\frac{\partial G}{\partial\vec{n}_m}\phi\phi_* d\Gamma$$

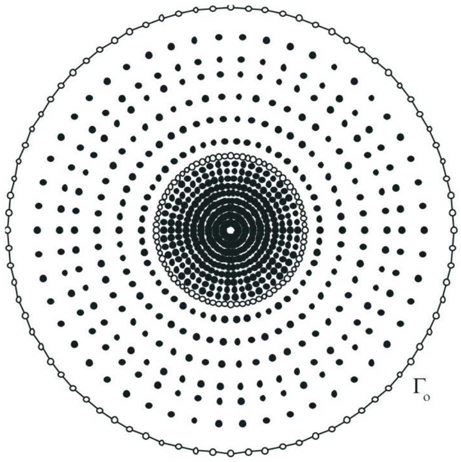

Figure 2. The distribution of boundary element nodes and internal collocation points used in the current GDRBEM.

$$= c_\xi^{(o)} G(\xi)\,\phi^I(\xi) - \int_{\Gamma_o} G\left(\phi^I q* - \phi * q^I\right) d\Gamma \\ + \sum_{j=1}^{N} \alpha_j [c_\xi^{(i,m)} \hat{\phi}_j(\xi) + \int_{\Gamma_o} (\hat{\phi}_j q* - \phi * \hat{q}_j) d\Gamma] \qquad (26)$$

The fundamental difference between equation (25) and equation (26) is that there is one more integral term along the top boundary Γ_i in equation (25). It is clear that the errors caused by the lack of this term cannot be patched up through the mesh refining near Γ_i.

As it will be shown in the next section, with this extra term being included in equation (25), the accuracy of our current GDRBEM model will be greatly improved.

NUMERICAL EXAMPLES AND DISCUSSION

In this section, using the current GDRBEM model, we shall present some numerical results of wave amplification for a submerged truncated conical shoal. In all calculations related to the current GDRBEM model, 40 quadratic boundary elements (with 80 boundary nodes) on both Γ_i and Γ_o , 300 internal collocation points in the middle domain Ω_m and 450 internal collocation points in the inner domain Ω_i , are always used (see

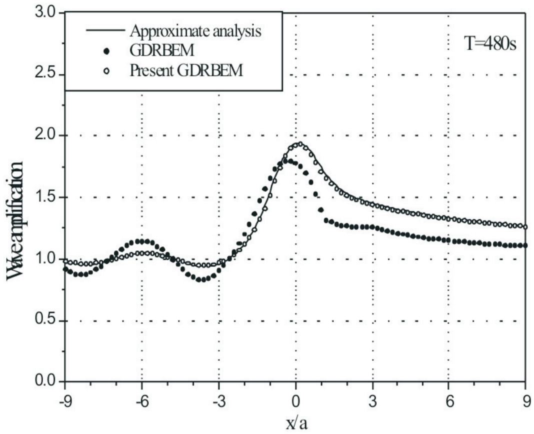

Figure 3. Wave amplifications along the x-axis for =480s calculated by the approximate analytic model (LIU *et al*., 2004), the GDRBEM model (ZHU *et al*., 2000) and the present GDRBEM model.

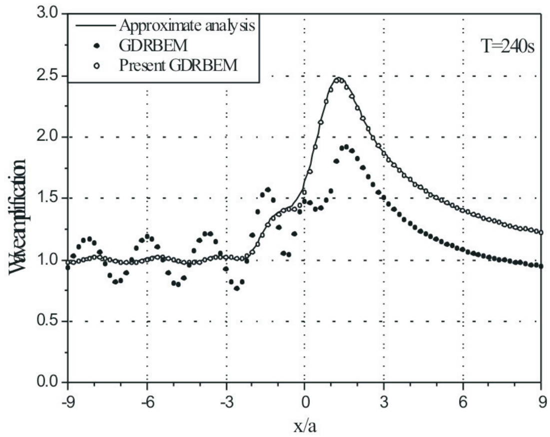

Figure 4. Wave amplifications along the x-axis for =240s calculated by the approximate analytic model (LIU *et al*., 2004), the GDRBEM model (ZHU *et al*., 2000) and the present GDRBEM model.

Figure 2). LIU *et al*. (2004) presented an approximate analytic solution of the MSE for wave scattering by a special submerged truncated conical shoal with the vertex being situated at the undisturbed free surface, where a=10km, b=30km, h_i=1.333km, h_o=4km and h'=0km, see Figure 1. Their results are ideal for testing our improved GDRBEM model.

The wave amplifications along the x-axis for two cases T=480s and T=240s are first calculated by using the approximate analytic model (LIU *et al*., 2004), the current improved GDRBEM model based on equation (25) and the GDRBEM model (ZHU *et al*., 2000) based on equation (26), respectively. In which, the total number and locations of all collocation points used in the GDRBEM model and in the current improved GDRBEM model are same. These three sets of results are presented in Figure 3 for T=480s and in Figure 4 for T=240s. As we can see, agreements between approximate analytic solutions and the present GDRBEM solutions for both cases are excellent, but significant discrepancies between the conventional GDRBEM solutions and approximate analytic solutions are observed, especially in the case T=240s. It is worth indicating that not much improvement on the accuracy of the GDRBEM model is obtained after we increase both boundary nodes and internal collocation points.

With the validation of the current GDRBEM model, it is then used to investigate the influence of shoal submergence on wave amplification. In LIU *et al*. (2004), to obtain an approximate

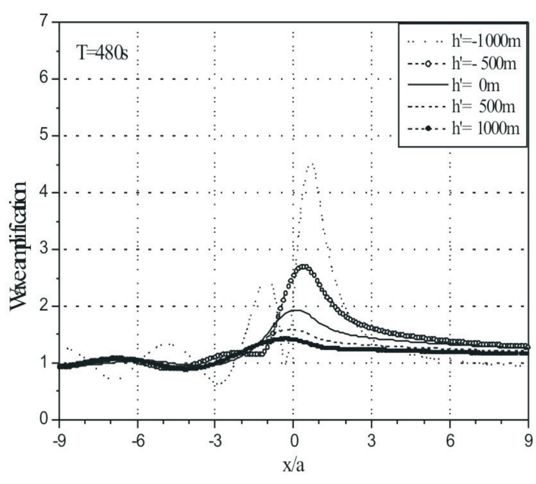

Figure 5. Wave amplifications along the x-axis for various submergences in the case of =480s.

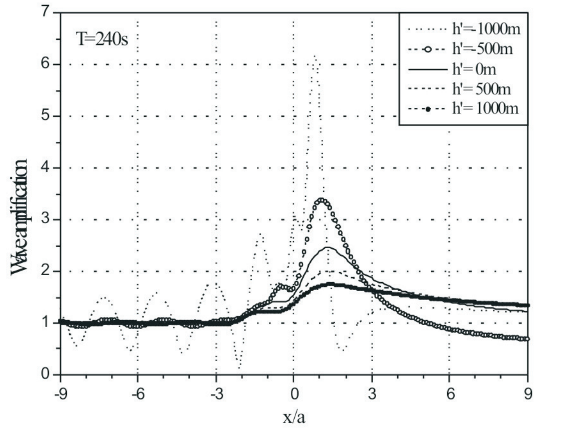

Figure 6. Wave amplifications along the x-axis for various submergences in the case of *T*=240s.

analytic solution of the MSE, the vertex of the conical shoal has to be located at the undisturbed free water surface. This restriction does not apply to the GDRBEM model. In this study, we let *h'* represent the distance of the vertex of the shoal from the still water surface. In two cases of *T*=480s and *T*=240s for the same truncated shoal studied above, wave amplifications along the x-axis are calculated for *h'* =-1.0km, -0.5km, 0km, 0.5km and 1.0km, and the results are presented in Figures 5-6, respectively. It is observed the maximal wave amplification decrease when the water depth increases as the impact from the shoal on the wave becomes weaker and weaker. On the other hand, it is interesting to observe that, for *T*=480s, the focal point moves upstream as the increase of water depth, however, for *T*=240s, the focal point moves downstream as the increase of water depth. The wave reflection becomes more evident for shorter wave (e.g. *T*=240s), which forms partially standing wave in front of the shoal when the local water depth is shallow.

CONCLUSIONS

In this paper, wave scattering by a submerged truncated shoal is modeled numerically. Considering that the gradient of the water depth is discontinuous along the boundary of the shoal top, we extended the conventional GDRBEM wave model (ZHU *et al*., 2000 and LIU, 2001), in which the computational domain was divided into two sub-domains along the footline, to an improved GDRBEM model with the computational domain being divided into three sub-domains where the inner region with constant depth is an individual sub-domain. This treatment leads to a new integral term appearing in the final boundary integral equation in the improved GDRBEM model. It is shown that, with this new term being included, the present GDRBEM model provides much more accurate results than the conventional GDRBEM model does.

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