Surface Roller of Breaking Waves at Barred Beaches

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ABSTRACT

A new methodology to determine the area of the surface roller of breaking waves is proposed and verified. The method is based on two existing undertow formulations and determines the cross-shore variation of the surface roller with the use of the vertical distribution of the undertow and of the time variation of the wave surface elevation. A new set of data from a large-scale laboratory experiment over a fixed-bed barred beach has been considered. For the plunging breaker case, the measured values of the cross-sectional area of the surface roller from the present method compare favorably with the calculated values from three existing formulae. For the spilling breaker case the calculated values capture the cross-bar variation of the surface roller with an overestimation. Both breaker types present a linear decay of the surface roller over the bar at transects past the breaking point.

INTRODUCTION

Surf zone dynamics is a highly complicated topic which requires a detailed knowledge of the breaking waves. In the last decades the interest for hydrodynamics of breaking waves at beaches has largely increased with the scope to describe sediment transport mechanisms and beach morphology. In particular, Svendsen (1984) pointed out that the in surf zone the presence of a surface roller is of importance when discussing the momentum and the energy equations. Any way, due to the difficulties to distinguish a possible roller area from the rest of the wave motion when using a wave gauge in laboratory experiments, the presence of a surface roller has been rarely taken into account. Svendsen (1984) posed \( A \approx 0.9H \) where \( A \) is the cross-sectional area of the surface roller and \( H \) is the wave height. Based on a hydraulic jump model of Engelund (1981), Deigaard and Fredsoe (1991) assumed that the front of a broken wave is similar to that of a bore or a hydraulic jump and approximated the cross-sectional area of the roller as \( A = 1.42H^2/h \) where \( h \) is the still water depth. Okayasu et al. (1986) considered the influence of the wave period and proposed an empirical formula for computing the cross-sectional area as \( A = kHL \) where \( L \) is the wave length and \( k \) is a coefficient ranging between 0.06 and 0.07.

The objective of the present work is to obtain, for the first time, the magnitude of the roller surface across a beach bar till in proximity of the wave reforming region under spilling and plunging breakers. A new methodology is proposed to determine the surface roller when the free surface elevation and the offshore return current or undertow measurements at a given transect are available. The method is based on a theoretical interpretation which considers the interaction between the mass of water carried landward by the breaking waves in a surf zone and the undertow. For this purpose a new set of data has been considered from a large-scale laboratory experiment over a fixed-bed barred beach at the wave flume of the Polytechnic University of Catalonia in Barcelona, Spain.

EXPERIMENTAL SET-UP AND DATA ANALYSIS

The large-scale laboratory experiment on wave hydrodynamics over a fixed-bed barred beach has been performed at 100 m long, 3 m wide and 5 m deep wave flume. Figure 1 shows the rigid bottom beach profile where \( z = 0 \) is the vertical coordinate with \( x = 0 \) at the still water level. The profile was designed to match an equilibrium bar. This was accomplished by scaling-down prototype profiles at Duck (North Carolina, USA) and taking into account the SUPERTANK (Kraus and Smith, 1994) and DELTA-flume (Sanchez-Arcilla et al., 1995) movable-bed experiments. Three types of monochromatic wave conditions (designated as tests 1, 2, and 3, respectively) were produced by a piston type wave maker equipped with a reflected-wave absorption system. Waves broke on the seaward slope of the bar and reformed into the trough region, breaking secondly nearer the shoreline. Table 1 reports the wave conditions characteristics, where \( h \) is the target regular wave height in front of the wave maker, \( T \) is the wave period, \( L \) is the computed wavelength at the wave-maker (using linear theory), \( x \), \( H \), and \( h \) are the approximate breaking location, breaking wave height (defined as the maximum wave height from wave height measurements) and depth, respectively. Waves 1 and 3 have approximately the same offshore wave steepness, and 1, 2 have the same offshore wave height, and 2 and 3 have the same wave period.

Seven spherical S-type electromagnetic current meters (ECM) with 8 Hz sampling frequency have been used. The ECM measurements cover the vertical range between the mean surface elevation and 15 cm above the bottom, every 5 cm apart. In addition to the ECM, two acoustic Doppler current meters (ADVLab by Nortek) have been used to measure the three-component flow velocities at both 25 and 50 Hz sampling frequencies. The ADVLab measurements were performed at eight different transects along the bar with a vertical ranging between 5 and 7 cm for 45 points.

The co-located free surface elevation and velocity measurements were carried out at some selected transects across the bar (Figure 2). All the surface elevation and velocity data samples were synchronized by a trigger signal. For repeatability and quality control of the tests, three surface elevation sensors remained the whole time of the experiment at fixed positions in front of the wave paddle. The wave conditions 1, 2, and 3 were run sequentially for each test repetition. Wave surface elevation and velocity measurements at each transect reflected over-10 repetitions of the same wave condition.

Measured Undertow Profiles

A minimum record length of 50 waves has been considered for calculating the time-mean from the measurements with the two different velocity meters at each test repetition. For all the ADVLab measurements, the level of the auto-correlation and of
the signal-to-noise ratio (SNR) levels has been assessed. For most of the sampled time series these resulted to be larger than 90 and 20 dB, respectively.

For wave 1, with wave breaking of spilling type at $x = 4050$ cm, the variation of the time mean flow, $\bar{u}$, with depth at several locations along the bar has been measured. The data (not shown here) present uncertainties at $x = 3850$ cm as the ADVLab and ECM results differ significantly. At all other sections, the measured mean horizontal velocities from both the ADVLab and ECM sensors present reasonable agreement and yield consistent undertow profiles. For wave condition 2 which formed a plunging breaker at $x = 4250$ cm, Figure 3 shows the measured variation of $\bar{u}$ with depth at the considered transects across the bar. Analogously to the results for wave 1 at $x = 3850$ cm, the ADVLab and ECM data in Figure 3 present a contradictory trend at $x = 4050$ cm. At all other sections, the measured mean horizontal velocities from both the ADVLab and ECM sensors present reasonable agreement and yield consistent undertow profiles.

For wave condition 2 which formed a plunging breaker at $x = 4250$ cm, Figure 3 shows the measured variation of $\bar{u}$ with depth at the considered transects across the bar. Analogously to the results for wave 1 at $x = 3850$ cm, the ADVLab and ECM data in Figure 3 present a contradictory trend at $x = 4050$ cm. At all other sections, the measured mean horizontal velocities from both the ADVLab and ECM sensors present reasonable agreement and yield consistent undertow profiles.

The comparison of undertow profiles from wave 1 and 2 indicates that the return current at each transect presents the same order of magnitude. Waves 3 and 2 where selected in order to induce plunging breakers. Larger undertow values for wave 3 (not shown here) with respect to those for wave 2, make evident the stronger influence of $H$ in comparison to that of the wave period in determining the order of magnitude of undertow and, consequently, of the surface roller area. It is found that the wave period influences the uniformity along the vertical.

**METHODOLOGY**

The detection and the measurement of the cross-sectional area of a surface roller may be typically conducted by means of video techniques. The proposed method allows to obtain estimates of the surface roller at a given transect based on measurements of the vertical distribution of the undertow and of the time variation of the surface elevation. In fact, when considering a two-dimensional scheme, the mass of water carried landward by the breaking waves in a surf zone has to be compensated by a seaward return flow, the undertow. In particular, SVENDSEN (1984) proposed the following relationship between the depth averaged velocity below the wave trough, $\bar{u}_b$, and $A$

$$\bar{u}_b = -c \left( \frac{H}{h_0} \right)^{\gamma} \left( B_u + \frac{Ah}{H^2L} \right)$$  \hspace{1cm} (1)

where $B_u = \frac{1}{T} \int \left( \frac{\eta}{H} \right)^2 dt$ is a wave shape factor ($=1/8$ for sinusoidal waves) with $\eta = $ wave surface elevation, and $c =$ wave celerity ($=\sqrt{gh}$ at shallow water conditions).

Values of $\bar{u}_b$ from the experimental investigation have been obtained as the ratio of the volume flux below the trough level, $Q$, to the water depth below the trough level, $d$. TOMASICCHIO and SANCHO (2002) shown that the present data further validate the kinematic undertow model proposed by COX and KOBAYASHI (1997, 1998) which assumes a parabolic profile for the interior layer and is capable of predicting the undertow profiles both inside and outside the surf zone. According to the kinematic model, the undertow from the top of the bottom boundary layer to the trough level is expressed as a 2nd order equation

$$\bar{u} = \bar{u}_b + \alpha z_h^2$$  \hspace{1cm} (2)

where $\bar{u}_b =$ hypothetical undertow velocity in the absence of the boundary layer; $\alpha =$ dimensional coefficient; $z_h =$ vertical coordinate above the bottom, positive upward with zero at the bottom. The values of $\bar{u}_b$ and $\alpha$ for the 2nd order equation have been estimated in order to have a graphical best fit through the (screened) measured undertow data. The curve is similar to that

<table>
<thead>
<tr>
<th>Wave condition</th>
<th>$H_s$ (m)</th>
<th>$T$ (s)</th>
<th>$H_s/L$</th>
<th>$x_c$ (m)</th>
<th>$H_s$ (m)</th>
<th>$h_s$ (m)</th>
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</thead>
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<tr>
<td>1</td>
<td>0.21</td>
<td>2.5</td>
<td>0.024</td>
<td>40.5</td>
<td>0.30</td>
<td>0.41</td>
</tr>
<tr>
<td>2</td>
<td>0.21</td>
<td>3.5</td>
<td>0.015</td>
<td>42.0</td>
<td>0.35</td>
<td>0.45</td>
</tr>
<tr>
<td>3</td>
<td>0.38</td>
<td>3.5</td>
<td>0.027</td>
<td>46.5</td>
<td>0.58</td>
<td>0.56</td>
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Figure 2. The considered transects across the bar.
Table 2. Volume flux and depth averaged velocity below the wave trough at the transects across the bar.

<table>
<thead>
<tr>
<th>Wave</th>
<th>x (cm)</th>
<th>Q_v (m³/s)</th>
<th>α (ms⁻¹)</th>
<th>u_b (m/s)</th>
<th>u_w (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3450</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3600</td>
<td>-0.0187</td>
<td>1.6</td>
<td>-0.13</td>
<td>0.0476</td>
<td>-</td>
</tr>
<tr>
<td>3850</td>
<td>-0.0133</td>
<td>1.6</td>
<td>-0.05</td>
<td>-0.655</td>
<td>-</td>
</tr>
<tr>
<td>4050</td>
<td>-0.0221</td>
<td>1.9</td>
<td>0</td>
<td>-0.674</td>
<td>-</td>
</tr>
<tr>
<td>4200</td>
<td>-0.0229</td>
<td>-1.7</td>
<td>-0.007</td>
<td>-0.693</td>
<td>-</td>
</tr>
<tr>
<td>4350</td>
<td>-0.0242</td>
<td>-1.2</td>
<td>-0.012</td>
<td>-0.567</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. Cross-shore variation of the still water depth, h, of the mean wave height, H, and of the wave shape factor, B_w.

<table>
<thead>
<tr>
<th>Wave</th>
<th>x (cm)</th>
<th>h (m)</th>
<th>H (m)</th>
<th>B_w (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3450</td>
<td>-0.45</td>
<td>0.255</td>
<td>0.0951</td>
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<tr>
<td>4200</td>
<td>0.41</td>
<td>0.239</td>
<td>0.0793</td>
<td></td>
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<tr>
<td>4050</td>
<td>0.393</td>
<td>0.274</td>
<td>0.0748</td>
<td></td>
</tr>
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<td>3850</td>
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<td>0.214</td>
<td>0.0844</td>
<td></td>
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<tr>
<td>3600</td>
<td>0.45</td>
<td>0.206</td>
<td>0.0884</td>
<td></td>
</tr>
<tr>
<td>3450</td>
<td>0.492</td>
<td>0.119</td>
<td>0.0925</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Wave B. Measured variation of the undertow across the bar; (+) ECM, (o) ADVLab. x in cm.
obtained using a quadratic polynomial fit through the data points. This resulted in a simplification of the kinematic undertow model, as the boundary layer logarithmic variation is not included, and the 2nd order data interpolation is forced to go through a given value at the bottom. Once \( Q \) has been calculated, values of \( \bar{n} \) have been found as \( \bar{n} = Q / d \). Table 2 lists the variation of \( \bar{n} \), \( \alpha \), and \( Q \) across the bar for the different wave conditions. Negative values of \( \alpha \) are found at transects where waves have not fully broken yet. For all wave conditions, the curvature coefficient appears to have a maximum of about 2.0 at some distance after the wave breaking. It takes values smaller than the maxima immediately after the breaking point and nearer the end of the surf zone. There also appears a certain correlation between \( \alpha \) and the geometrical characteristics at the considered transect. As expected, \( \bar{n} \) attains the largest values at the transects close to the breaking point.

At a given transect, a zero-crossing analysis of the time variation of the free surface elevation has been conducted for each of the repetitions of the same wave condition. The mean value of the wave height, \( H \), and the mean value of the wave shape factor, \( B \), have been obtained representing all the repetitions and a certain wave condition at the considered transect. Table 3 summarizes the results of the wave surface elevation analysis. Values of \( B \) agree with the pattern proposed by BASCO and YAMASHITA (1986) and HANSEN (1990) giving larger values of the wave shape factor for the spilling breaker compared to the plunging one. The lower \( B \) values are attained in proximity of the breaking point. Furthermore, the \( B \) values show an influence from the decreasing bottom slope throughout the surf zone past the bar. In particular, they increase at the transects past the bar crest approaching the wave reforming region.

**DISCUSSIONS**

Figure 4 shows the values of the surface roller, at the considered transects for the three adopted wave conditions, which have been determined by the present method (measured) and by the formulae from three different Authors (calculated). For wave 1, giving a spilling breaker, the measured \( A \) values present a linear increase from almost zero at the breaking point (0.0266 m\(^2\) at \( x = 40.5 \) m) with a decay when past the bar crest. For wave 2, the breaker determines the largest \( A \) value through the bar when at the plunging point. The surface roller value at the plunging point of wave 3 (\( x = 46.5 \) m) is not shown, but the figure suggests that it is larger than for the wave 2 case. This indicates also the larger influence of the wave height on the size of the surface roller for a given wave period. All breaker types induce an almost linear decay of \( A \) over the bar at transects past the breaking point. Furthermore, it can be noticed that \( A \) becomes negligible when approaching the wave reforming region which has been found to be located at \( x = 32 \) m. There

![Figure 4. Measured and calculated surface roller values at the considered transects: (a) wave 1; (b) wave 2; (c) wave 3. () experimental, (?) Svendsen's formula, (?) Deigaard and Fredsøe's formula, (?) Okayasu's formula.](image-url)
also appears the correlation between $A$ and $\bar{u}_w$, when considering that both of them present the largest values at the transects close to the breaking point and that have a similar cross-bar variation.

The values of the area of the surface roller at the considered transects for all wave conditions have been calculated with three different formulae and are shown in Figure 4. Their comparison with the measured values indicates that the Okayasu's formula gives the poorest agreement. This is almost certainly due to the wave length that, in the lack of the experimental value, has been calculated with the linear theory. For the spilling breaker case, all the considered formulae give results systematically larger than the measured values. For the two considered plunging breaker cases, the Svendsen (1984) and Deigaard and Fredsøe (1991) formulae present a great accuracy in describing the variation of $A$ across the entire bar. The results from these two formulae appear very close to each other for both spilling and plunging breakers. This may be explained in view of the breaking criteria $H=0.78h$ (Dean and Dalrymple, 1984).

CONCLUSIONS

The present contribution introduces and verifies a new methodology to measure the area of the surface roller of breaking waves when the vertical distribution of the undertow and the time variation of the surface elevation are known. The method is based on two existing undertow formulations and allows to obtain information on the cross-shore variation of the surface roller with the use of instruments which are common to most coastal laboratories.

Based on a quasi field experimental investigation, the method makes available a new set of measured values of $A$. For a spilling breaker, the surface roller $A$ shows a linear increase from almost zero till the breaking point. Both spilling and plunging breakers present a linear decay of $A$ over the bar at transects past the breaking point till the wave reforming region.

The values of $A$ which have been measured with the present method show a great agreement with the calculated ones for the plunging breaker case. A discrepancy is observed for the spilling breaker. Finally, the comparison indicates that the use of the considered formulae may be extended to the case of plunging breakers over a barred beach.

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LITERATURE CITED


